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Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, October 2016
(2013 Scheme)
13.303 : DISCRETE STRUCTURES (FR)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **2** marks :

1. State the converse and contrapositive of each of the following :
   a) If it rains, I am not going
   b) I can't complete the task, if I don't get more help.
2. Without using truth table, prove that

   $p \to (q \vee r) \Leftrightarrow (p \to q) \vee (\neg p \vee r)$.
3. Symbolize the statement : Every train is faster than some cars.
4. Prove that the set of integers is denumerable.
5. Find fog and gof when $f : R \to R$ and $g : R \to R$ defined by $f(x) = 2x - 1$, $g(x) = x^2 - 2$.
6. Let R be the relation in the natural numbers N defined by $R = \{(x, y) \mid x \in N, y \in N,$ $(x - y)$ is divisible by 5$\}$. Prove that R is an equivalence relation.
7. What is a monoid ? Give an example.
8. Prove that the identity element in a group is monoid.
9. What is a complete graph ? Give an example.
10. What do you mean by a bounded lattice ?

P.T.O.

PART – B

Answer **one full** question from **each** module. **Each** question carries **20** marks :

Module – I

11. a) Prove the validity of the following arguments. Every living thing is a plant or an animal. David's dog is alive and it is not a plant. All animals have hearts. Hence, David's dog has a heart. **10**

b) Show that $\forall x\,[P(x) \vee Q(x)] \Rightarrow \forall x\,P(x) \vee \exists x\,Q(x)$ using indirect method of proof. **10**

OR

12. a) Determine the validity of the following arguments.

If today is Tuesday, then I have a test in Computer Science or a test in Economics. If my Economics professor is sick, then I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore I have a test in Computer Science. **12**

b) Derive using CP rule.

$P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$. **8**

Module – II

13. a) Show that the function $f : R \rightarrow R$ defined by $f(x) = 3x - 1$, $x \in R$ is a bijection. **5**

b) Show that if any 11 numbers are chosen from the set {1, 2, ...20}, then one of them will be a multiple of another. **5**

c) Prove by mathematical induction $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ **5**

d) Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations ? **5**

OR

14. a) Let $S = \{1, 2, 3, 4\}$ and A be $S \times S$. Define the following relation R on A as (a, b) R (a′, b′) if and only if $a + b = a' + b'$.

i) Show that R is an equivalence relation.

ii) Compute A/R. **10**

b) Explain Piano Axioms. 5

c) Determine whether the following relation is a partial order on set A. Give reasons for your answer A = I (set of Integers), aRb if and only if a = 2b. 5

Module – III

15. a) State and prove Lagrange's theorem. 10

b) Prove that the set Q of all rational numbers other than 1 with the operations defined by $a*b = a + b - ab$ constitute on abelian group. 10

OR

16. a) Let (H, .) be a subgroup of a group (G, .). Let $N = \{x | x \in G, xHx^{-1} = H\}$. Show that (N, .) is a subgroup of (G, .). 10

b) Let (A, $*$) be a group. Show that (A, $*$) is an abelian group if and only if $a^2 * b^2 = (a * b)^2$ for all a and b on A. 5

c) Prove that every group of prime order is cyclic. 5

Module – IV

17. a) Prove that the zero element and the unity element of a Boolean algebra B are unique. 5

b) Discuss the different properties of a lattice. 10

c) Differentiate between a complete graph and a connected graph with examples. 5

OR

18. a) Simplify the following boolean expression $((a \wedge b) \vee c) \wedge ((a \vee b) \wedge c)$. 5

b) Differentiate between connected, disconnected, weakly connected and strongly connected graphs with examples. 10

c) Prove that in a distributive lattice, if $b \wedge c = 0$, then $b \leq c$. 5
